Lineare DGL-Systeme

Stufe 3: Anfangswertproblem

Name: _____ Datum: _____

Löse das Anfangswertproblem und gib die gesuchte Koordinate als Funktion von t an.

1) $\vec{x}' = \begin{pmatrix} -2 & 0 \\ -4 & 2 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} -2 \\ -1 \end{pmatrix}: \quad y(t) = ?$ _____

2) $\vec{x}' = \begin{pmatrix} -2 & 4 \\ 0 & 2 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 0 \\ -2 \end{pmatrix}: \quad x(t) = ?$ _____

3) $\vec{x}' = \begin{pmatrix} -3 & 0 \\ 6 & 3 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}: \quad x(t) = ?$ _____

4) $\vec{x}' = \begin{pmatrix} 2 & 5 \\ 0 & -3 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}: \quad x(t) = ?$ _____

5) $\vec{x}' = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}: \quad x(t) = ?$ _____

6) $\vec{x}' = \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}: \quad x(t) = ?$ _____

7) $\vec{x}' = \begin{pmatrix} 2 & -2 \\ 0 & 0 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}: \quad y(t) = ?$ _____

8) $\vec{x}' = \begin{pmatrix} -3 & 5 \\ 0 & 2 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} -3 \\ 3 \end{pmatrix}: \quad x(t) = ?$ _____

9) $\vec{x}' = \begin{pmatrix} -2 & 6 \\ 0 & 1 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} -2 \\ -2 \end{pmatrix}: \quad x(t) = ?$ _____

10) $\vec{x}' = \begin{pmatrix} 3 & 5 \\ 0 & -2 \end{pmatrix}\vec{x}, \quad \vec{x}(0) = \begin{pmatrix} -2 \\ 0 \end{pmatrix}: \quad x(t) = ?$ _____

Lösungen: Lineare DGL-Systeme

Stufe 3: Anfangswertproblem

1) $\lambda_1 = 2,\ \vec{v}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix};\ \lambda_2 = -2,\ \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix};\ C_1 = 1,\ C_2 = -2:\quad y(t) = e^{2t} - 2e^{-2t}$

2) $\lambda_1 = -2,\ \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \lambda_2 = 2,\ \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix};\ C_1 = 2,\ C_2 = -2:\quad x(t) = 2e^{-2t} - 2e^{2t}$

3) $\lambda_1 = -3,\ \vec{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix};\ \lambda_2 = 3,\ \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix};\ C_1 = -1,\ C_2 = 1:\quad x(t) = -e^{-3t}$

4) $\lambda_1 = -3,\ \vec{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix};\ \lambda_2 = 2,\ \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ C_1 = -1,\ C_2 = 3:\quad x(t) = -e^{-3t} + 3e^{2t}$

5) $\lambda_1 = 2,\ \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix};\ \lambda_2 = 3,\ \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ C_1 = 3,\ C_2 = -4:\quad x(t) = 3e^{2t} - 4e^{3t}$

6) $\lambda_1 = -2,\ \vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix};\ \lambda_2 = -3,\ \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ C_1 = 1,\ C_2 = 0:\quad x(t) = 2e^{-2t}$

7) $\lambda_1 = 2,\ \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \lambda_2 = 0,\ \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix};\ C_1 = 0,\ C_2 = 1:\quad y(t) = 1$

8) $\lambda_1 = 2,\ \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix};\ \lambda_2 = -3,\ \vec{v}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix};\ C_1 = 3,\ C_2 = 6:\quad x(t) = 3e^{2t} - 6e^{-3t}$

9) $\lambda_1 = -2,\ \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \lambda_2 = 1,\ \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix};\ C_1 = 2,\ C_2 = -2:\quad x(t) = 2e^{-2t} - 4e^{t}$

10) $\lambda_1 = 3,\ \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \lambda_2 = -2,\ \vec{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix};\ C_1 = -2,\ C_2 = 0:\quad x(t) = -2e^{3t}$