Laplace-Transformation

Stufe 3: Anfangswertproblem

Name: _____ Datum: _____

Löse das Anfangswertproblem mit der Laplace-Transformation.

1) $y'' + 3y' + 2y = 0, \quad y(0) = 4,\ y'(0) = -7$ _____

2) $y'' + y' - 6y = 0, \quad y(0) = 0,\ y'(0) = -15$ _____

3) $y'' - y' - 2y = 0, \quad y(0) = -1,\ y'(0) = -5$ _____

4) $y'' - y' - 2y = 0, \quad y(0) = 5,\ y'(0) = 4$ _____

5) $y'' - 4y' + 3y = 0, \quad y(0) = -6,\ y'(0) = -12$ _____

6) $y'' + 3y' + 2y = 0, \quad y(0) = -3,\ y'(0) = 4$ _____

7) $y'' - y = 0, \quad y(0) = -2,\ y'(0) = -4$ _____

8) $y'' - y' = 0, \quad y(0) = 0,\ y'(0) = 3$ _____

9) $y'' - 4y = 0, \quad y(0) = 0,\ y'(0) = -12$ _____

10) $y' + 2y = -8, \quad y(0) = -1$ _____

11) $y'' - 9y = 0, \quad y(0) = 0,\ y'(0) = 6$ _____

12) $y'' + 4y' + 3y = 0, \quad y(0) = 6,\ y'(0) = -12$ _____

13) $y'' + 2y' = 0, \quad y(0) = -2,\ y'(0) = 6$ _____

14) $y'' - 3y' = 0, \quad y(0) = -2,\ y'(0) = 3$ _____

Lösungen: Laplace-Transformation

Stufe 3: Anfangswertproblem

1) $Y(s) = \frac{4s+5}{(s+2)(s+1)} = \frac{3}{s+2} + \frac{1}{s+1} \Rightarrow y(t) = 3e^{-2t} + e^{-t}$

2) $Y(s) = \frac{-15}{(s+3)(s-2)} = \frac{3}{s+3} - \frac{3}{s-2} \Rightarrow y(t) = 3e^{-3t} - 3e^{2t}$

3) $Y(s) = \frac{-s-4}{(s+1)(s-2)} = \frac{1}{s+1} - \frac{2}{s-2} \Rightarrow y(t) = e^{-t} - 2e^{2t}$

4) $Y(s) = \frac{5s-1}{(s+1)(s-2)} = \frac{2}{s+1} + \frac{3}{s-2} \Rightarrow y(t) = 2e^{-t} + 3e^{2t}$

5) $Y(s) = \frac{-6s+12}{(s-1)(s-3)} = -\frac{3}{s-1} - \frac{3}{s-3} \Rightarrow y(t) = -3e^{t} - 3e^{3t}$

6) $Y(s) = \frac{-3s-5}{(s+2)(s+1)} = -\frac{1}{s+2} - \frac{2}{s+1} \Rightarrow y(t) = -e^{-2t} - 2e^{-t}$

7) $Y(s) = \frac{-2s-4}{(s+1)(s-1)} = \frac{1}{s+1} - \frac{3}{s-1} \Rightarrow y(t) = e^{-t} - 3e^{t}$

8) $Y(s) = \frac{3}{s(s-1)} = -\frac{3}{s} + \frac{3}{s-1} \Rightarrow y(t) = -3 + 3e^{t}$

9) $Y(s) = \frac{-12}{(s+2)(s-2)} = \frac{3}{s+2} - \frac{3}{s-2} \Rightarrow y(t) = 3e^{-2t} - 3e^{2t}$

10) $sY - (-1) + 2Y = \frac{-8}{s} \Rightarrow Y(s) = -\frac{4}{s} + \frac{3}{s+2} \Rightarrow y(t) = -4 + 3e^{-2t}$

11) $Y(s) = \frac{6}{(s+3)(s-3)} = -\frac{1}{s+3} + \frac{1}{s-3} \Rightarrow y(t) = -e^{-3t} + e^{3t}$

12) $Y(s) = \frac{6s+12}{(s+3)(s+1)} = \frac{3}{s+3} + \frac{3}{s+1} \Rightarrow y(t) = 3e^{-3t} + 3e^{-t}$

13) $Y(s) = \frac{-2s+2}{(s+2)s} = -\frac{3}{s+2} + \frac{1}{s} \Rightarrow y(t) = -3e^{-2t} + 1$

14) $Y(s) = \frac{-2s+9}{s(s-3)} = -\frac{3}{s} + \frac{1}{s-3} \Rightarrow y(t) = -3 + e^{3t}$